

Interaction of Flow Pulsations and Chemical Reaction in Turbulent Flames

V.B. Librovich* and V.I. Lisitzyn†
USSR Academy of Sciences, Moscow, USSR

A general equation of the balance of turbulence energy in reacting flows is applied to analyze the interaction of the chemical reaction and turbulent velocity fluctuations. A flame in the small-scale turbulent field is considered, where the influence of the turbulence is displayed by the intensification of the heat and mass transfer processes in the laminar-similar front and by the increase in the effective chemical reaction rate. The interchange of the energy between the turbulent motion and the mean gas flow, together with the work providing the turbulent mass transfer in the field of the accelerated mean gas flow, appears to diminish the kinetic energy of the turbulent motion. The decay of turbulence is more pronounced for the high-temperature flames and the flows with the initial low level of turbulence. This effect, which influences the turbulent flame propagation rate, depends on the scale of turbulence, and decreases when it increases.

Nomenclature

a	= concentration of reacting component
c_p	= heat capacity at constant pressure
$\bar{E} = \rho \overline{u_i' u_i'}/2$	= mean energy of turbulence per unit volume
E	= activation energy of chemical reaction
$\bar{e} = \overline{u_i' u_i'}/2$	= mean energy of turbulence per unit mass
\bar{F}	= influx of turbulence energy from the tube
$f_1(\alpha), f_2(\alpha)$	= dimensionless functions
k	= coefficient of proportionality in the chemical reaction rate expression
l	= characteristic scale of turbulence
n	= order of chemical reaction,
n_1, \dots, n_4	= dimensionless coefficients of proportionality of the order unity included in Prandtl relations
$P_e = \bar{u}_0 t_l / x_f$	= eigenvalue of the flame propagation equation
$P_2 = \bar{u}_0 x_2 / l \bar{e}_0^{1/2}$	= Peclet number in the chemical reaction zone
p	= gas pressure
q	= heat of reaction
R	= universal gas constant
s	= $4\epsilon_0/\alpha = 4\bar{e}_0^{1/2}/\alpha v_0$
s	= dimensionless parameter
T	= temperature
t	= time
t_f	= $\bar{\rho}_0/k\rho_0^b \exp(-E/RT_f)$
t_f	= characteristic time of chemical reaction at final temperature and initial density and concentration of reacting component
u_j	= gas velocity components
v_0	= velocity of turbulent flame propagation evaluated without taking into account the change of the turbulence energy in the flame and the influence of the temperature pulsations upon the mean chemical reaction rate
x_j	= space coordinates
x_f	= $(\bar{e}_0^{1/2} t_f)^{1/2}$ — characteristic length given by the equation of the heat conduction

x_1, x_2	= dimensionless preheated zone and chemical reaction zone, correspondingly
α	= \bar{T}_f/\bar{T}_0 = ratio of the final and initial temperatures,
β	= $R\bar{T}_f/E$, $\gamma = l/x_f$
δ_{ij}	= Kroneker symbol, $\delta_{ij} = 1, i=j; \delta_{ij} = 0, i \neq j$
$\epsilon = \bar{e}_0^{1/2}/\bar{u}_0$	= intensity of turbulence in the upstream flow
$\epsilon_v = \bar{e}_0^{1/2}/\bar{v}_0$	= intensity of turbulence corresponding to the burning rate v_0
Θ	= $E(T - \bar{T}_f)/R\bar{T}_f^2$; dimensionless instant temperature
θ	= $E(\bar{T} - \bar{T}_f)/R\bar{T}_f^2$; dimensionless mean temperature
θ_0	= $E(\bar{T}_f - \bar{T}_0)/R\bar{T}_f^2$
σ	= molecular temperature diffusivity
ρ	= gas density
Φ	= chemical reaction rate
$\varphi(\Theta)$	= $\alpha^{-1} [\Theta ^n / (1 + \beta\Theta)] \exp[\Theta/(1 + \beta\Theta)]$
$\varphi(\Theta)$	= instant value of the dimensionless chemical reaction rate at the temperature Θ

Indices

f	= burned gases
o	= initial flow
i, j	= sum indices
$(\cdot), < >$	= calculation of mean values
$(\cdot)'$	= fluctuation
$(\cdot)_*$	= the boundary between the preheated zone and the chemical reaction zone

Introduction

DURING the past years, theoretical and experimental investigations have been performed to help understand and describe the interaction of the exothermic chemical reaction and turbulent flow.¹⁻⁹ In some experiments,³⁻⁶ a mechanism of the generation of turbulence energy by burning has been proposed which assumes the existence of the wrinkled laminar flame front producing the nonuniform flowfield. Part of its kinetic energy goes to the turbulent motion. Other authors^{7,8} considered a volumetric mechanism of the interaction, and proposed a criterion of the generation of the turbulence energy, starting from the Rayleigh condition of the thermokinetic oscillations.¹⁰ This paper presents an analysis performed on the basis of the general equation of the balance of the turbulence energy in the reacting flows.¹¹

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*Head of Thermogasdynamic Laboratory, Institute for Problems in Mechanics.

†Research Scientist of Institute for Problems in Mechanics.

Equation for Turbulence Energy

Starting with the equations of momentum and mass conservation for the compressible viscous gas, one can deduce an equation of the balance of the turbulence energy per unit volume \bar{E}^{11-13}

$$\frac{\partial \bar{E}}{\partial t} + \frac{\partial}{\partial x_j} \left[\bar{E} \bar{u}_j + \overline{Eu'_j} \right] = -\bar{\rho} \overline{u'_j u'_j} \frac{\partial \bar{u}_i}{\partial x_j} - \bar{\rho}' \overline{u'_j u'_j} \frac{\partial \bar{u}_i}{\partial x_j} - \sigma'_{ij} \frac{\partial u'_i}{\partial x_j} + \left\{ \frac{\partial}{\partial x_j} \left[\overline{u'_j \sigma'_{ij}} - \bar{\rho}' u'_j \right] + \bar{\rho}' \frac{\partial u'_i}{\partial x_i} \right\} \quad (1)$$

The expressions in the square brackets on the left- and right-hand sides of Eq. (1) describe the spatial redistribution of the turbulence energy provided by the mean convective flow ($\bar{E} \bar{u}_j$), by the turbulent diffusion ($\overline{Eu'_j}$), by the pulsations of the viscous tension tensor ($\overline{u'_j \sigma'_{ij}}$), and by the pulsations of pressure $\bar{\rho}' u'_j$.

It should be noted that the first term of the right-hand side, which describes the interchange of the energy between the turbulent motion and the mean gas flow, is governed essentially by the kinematic properties of the mean flow. For instance, in the one-dimensional steady-state turbulent flame, the exothermic chemical reaction leads to the acceleration of the mean flow, and therefore a part of the turbulence energy passes into the kinetic energy of the mean movement (i.e.

$$-\bar{\rho} \overline{u'_j u'_j} \frac{d\bar{u}_i}{dx} < 0$$

to be definite, here and in the following, the gas flow is assumed to be directed to the side of positive $x_i \equiv x$).

The second term in the right-hand side of Eq. (1) is related to the work on turbulent mass transfer in the accelerated field of the averaged flow. Since pulsations of density in turbulent flows with combustion are determined mainly by temperature pulsations, the positive pulsation of velocity bringing a cooler gas into the point in question, produces a positive pulsation of density. Therefore, in the simplest case of one-dimensional steady-state turbulent flow, this term also plays the role of sink for turbulence energy

$$-\bar{\rho}' \overline{u'_j u'_j} \frac{d\bar{u}_i}{dx} < 0$$

The third term in the right-hand side of Eq. (1) describes the dissipation of the pulsation motion energy due to the forces of molecular viscosity. The last term which characterizes work of pressure pulsations during thermal expansion (compression) in the gas, is not vanishing only in a compressible gas. If there is no chemical reaction leading to thermal expansion of the gas, positive pressure pulsations ($\bar{p}' > 0$) result in gas compression ($\partial u'_j / \partial x_j < 0$); therefore, the resultant sign of this term is negative. If, however, an exothermic reaction takes place in the gas, positive pressure pulsations may increase the reaction rate, whereas negative ones reduce it, and thus modify the expansion (compression) of the gas correspondingly; as a result, the sign of the term $\bar{\rho}' \partial u'_j / \partial x_j$ may be changed to positive, which corresponds to a source of pulsation energy.

We have demonstrated earlier¹⁴ that this effect becomes appreciable at pulsation velocities comparable to that of sound; therefore, considering a turbulent flame with pulsation velocities much smaller than that of sound, we shall neglect generation of turbulence due to the work of pressure pulsations during thermal expansion of the gas.

We will rescript further discussion by the case of well-developed turbulence in which the transfer processes of turbulence energy, mass, and heat by velocity pulsations are much more intensive than molecular processes of transfer; therefore, we shall neglect the term $\overline{u'_j \sigma'_{ij}}$ in Eq. (1), as well as molecular diffusion and molecular heat conductivity in diffusion and heat conductivity equations (see the following).

We shall assume that turbulence of the flow entering the flame is isotropic, and that interaction between the flame and turbulence does not result in any appreciable anisotropy. Under this assumption, we can neglect the correlation $\bar{\rho}' u'_j$.¹²

Equation (1) will be applied further in the following to a steady-state, one-dimensional on the average, turbulent flame with a steady-state turbulent upstream flow. In order to insure the steady-state turbulence of the initial flow, dissipation of kinetic energy of turbulent motion must be compensated by either energy inflow from the region of turbulence initiation, or by turbulence generation on gradients of averaged flow velocity. The former case is observed for turbulent flow in channels, when turbulence generated in the boundary layer at the walls diffuses into the flow core; the latter case is realized in the turbulent boundary layer at the wall, where turbulence is generated on the logarithmic profile of the mean velocity.

In the case of a flow in a channel (pipe), there exists a profile of turbulent kinetic energy over the radius, leading to diffusion and providing the compensation of dissipation. If, at the same time, interaction between turbulence and flame produces variation of turbulence kinetic energy along the channel axis, the field of turbulent motion energy apparently is described by a two-dimensional equation of turbulence balance.¹ However, later in this article we shall use the one-dimensional equation obtained from Eq. (1) by averaging it over the radius

$$\frac{d}{dx} [\bar{E} \bar{u} + \overline{Eu'}] = -\bar{\rho} \bar{u}'^2 \frac{d\bar{u}}{dx} - \bar{\rho}' \bar{u}' \frac{d\bar{u}}{dx} + \left(\bar{F} - \sigma'_{ij} \frac{\partial u'_i}{\partial x_j} \right) \quad (2)$$

where u , u' are mean and pulsation components of velocity along the channel axis (x axis), $\bar{F} = \overline{Eu'}/\mathfrak{D}$ is the turbulent kinetic energy flux from walls, u'_i is the pulsation component of velocity along the radius, and \mathfrak{D} is the channel diameter.

Upstream of the flame, the inflow of turbulent motion energy and its dissipation compensate one-another, so that the expression in parentheses in Eq. (2) becomes zero. For the sake of simplification, we shall neglect, in further discussion, the lack of balance between energy inflow and turbulent motion energy dissipation, caused by modification of the latter in the flame. This simplification is applicable because of a small difference in the initial and final turbulence levels.

Let us analyze in more detail the physical meaning of mechanisms of flame action on turbulence caused by energy exchange between the averaged and pulsation motion, and by the work providing the turbulent mass transfer in the accelerated field of the averaged flow. An average pressure difference, $-d\bar{p}/dx$, generated in the flame, cancels out the inertia force in the averaged motion $\bar{p} \bar{u} \frac{d\bar{u}}{dx}$. However, elements of gas participate in the mean motion only on the average, deviating from it because of velocity and density pulsations; this generates additional inertia forces against which work needs to be done by turbulence.

Indeed, let us assume that elements of gas have density equal to the average density and that velocity pulsates around the average value $u = \bar{u} + u'$. The inertia force produced in this motion is now equal to

$$\bar{\rho} (\bar{u} + u') \frac{d\bar{u}}{dx}$$

and is not balanced any longer by the average pressure difference

$$-d\bar{p}/dx = \bar{\rho} \bar{u} \frac{d\bar{u}}{dx}$$

therefore, the resultant force appears,

$$f' = \bar{\rho} \bar{u} \frac{d\bar{u}}{dx} - \bar{\rho} (\bar{u} + u') \times \frac{d\bar{u}}{dx} = -\bar{\rho} u' \frac{d\bar{u}}{dx}$$

directed against the pulsation velocity ($d\bar{u}/dx > 0$ in the flame), thus impeding the pulsation motion. At the pulsation velocity u' , the work

$$f'u' = -\bar{\rho} u' u' d\bar{u}/dx$$

with nonzero average value

$$\overline{f'u'} = -\overline{\rho u' u'} d\bar{u}/dx$$

is done against this force at the expense of turbulent kinetic energy.

In a similar way, if density is the only oscillating quantity, additional force

$$f' = \bar{\rho} \bar{u} d\bar{u}/dx - (\bar{\rho} + \rho') \bar{u} \times d\bar{u}/dx = -\rho' \bar{u} d\bar{u}/dx$$

appears, also directed against the pulsation velocity, since in flames the density pulsations, generated by velocity pulsations, usually are in phase with them. This force impedes pulsation motion, and thus, at the velocity u' , the work is done against this motion at the expense of turbulent kinetic energy, with nonzero mean value

$$\overline{f'u'} = -\overline{\rho' u' \bar{u}} d\bar{u}/dx$$

Equations of Turbulent Flame

In order to calculate coefficients at correlations in Eq. (2), let us write the averaged equation for heat conductivity of compressible gas:

$$(d/dx) [\bar{\rho} \bar{u} \bar{T} + \bar{\rho} \bar{u}' \bar{T}' + \bar{T} \bar{\rho}' \bar{u}' + \bar{T} \bar{\rho}' \bar{u}' + \bar{u} \bar{T}' \bar{\rho}'] = q \bar{\Phi} C_p \quad (3)$$

and averaged continuity equation

$$\bar{\rho} \bar{u} + \bar{\rho}' \bar{u}' = \bar{\rho}' \bar{u}_0 \quad (4)$$

and the equation of gas state, written for instantaneous values of thermodynamical parameters

$$\bar{\rho} \bar{T} + \rho' T' + \bar{\rho} T' + \bar{T} \rho' = \bar{T}_0 \bar{\rho}_0 \quad (5)$$

Here C_p is gas specific heat at constant pressure, T is temperature, q is the heat of reaction, Φ is the average value of the reaction rate, which depends on temperature and concentration of the reacting component a . The pressure in Eq. (5) can be considered to be constant. Here and in the following, the subscript zero denotes the parameters of the initial turbulent flow. Equations (3 and 4) are written, for the simplest case of one-dimensional and steady-state turbulent flame.

From the heat conductivity [Eq. (3)], one can derive by use of the equation of gas state [Eq. (5)], the equation determining acceleration of the average flow. This requires two averaging operations: first we average Eq. (5)

$$\bar{\rho} \bar{T} + \bar{\rho}' \bar{T}' = \bar{\rho}_0 \bar{T}_0 \quad (6)$$

and then the equation obtained by multiplying Eq. (5) by u'

$$\bar{T} \bar{\rho}' \bar{u}' + \bar{\rho} \bar{u}' \bar{T}' + \bar{\rho}' \bar{u}' \bar{T}' = 0 \quad (7)$$

Simplifying Eq. (3) by means of Eqs. (6 and 7), we obtain

$$C_p \bar{\rho}_0 \bar{T}_0 d\bar{u}/dx = q \bar{\Phi} \quad (8)$$

According to Eq. (8), acceleration of the averaged flow is caused by heating up the gas by the heat released by the chemical reaction, and is not directly related to the turbulence heat conductivity. As a result, the velocity of the averaged flow in the preheated zone of the flame remains constant, despite the reduction of the averaged gas density; the increase

of the velocity of the averaged flow takes place in the reaction zone only. This feature occurs because reduction of the averaged density, caused by turbulence heat conductivity, leads to generation of a turbulent mass flux and, consequently, to reduction of the average convective mass flux $\bar{\rho} \bar{u}$. Therefore, the averaged velocity remains constant.

In order to obtain the equation determining distribution of the averaged temperature, we substitute in Eq. (3), according to the continuity equation [Eq. (4)], the sum of convective and turbulent mass fluxes $\bar{\rho} \bar{u} + \bar{\rho}' \bar{u}'$ by the initial mass flux $\bar{\rho}_0 \bar{u}_0$

$$\bar{\rho}_0 \bar{u}_0 \frac{d\bar{T}}{dx} = -\frac{d}{dx} (\bar{\rho} \bar{u}' \bar{T}') + \frac{q}{C_p} \bar{\Phi} - \frac{d}{dx} (\bar{u} \bar{\rho}' \bar{T}' + \bar{\rho}' \bar{u}' \bar{T}') \quad (9)$$

In order to calculate the distribution of the averaged concentration, we use the fact that, in the case of equal coefficients of molecular diffusion and molecular temperature conductivity, equations of heat conductivity and diffusion yield the similarity of instantaneous (hence of averaged) fields of concentration and temperature¹⁵

$$(\bar{T}_f - \bar{T}) / (\bar{T}_f - \bar{T}_0) = \bar{a} \quad (10)$$

where $\bar{T}_f = \bar{T}_0 + q/C_p$ is the averaged combustion temperature.

The set of equations (2, 4, 6, 8, and 9) is not complete with respect to the variables \bar{E} , $\bar{\rho}$, \bar{u} , and \bar{T} , because it contains unknown correlations between hydrodynamical and thermodynamical parameters. In order to complete the system, we use the hypotheses that can be regarded as a linear expansion of correlations into a series over the gradients of average values¹²

$$\bar{E} u' \approx \bar{\rho} \bar{u}' = -n_1 l \bar{e}^{1/2} d\bar{e}/dx \quad (11)$$

$$\bar{\rho}' \bar{u}' = -n_2 l \bar{e}^{1/2} d\bar{\rho}/dx \quad (12)$$

$$\bar{T}' \bar{u}' = -n_3 l \bar{e}^{1/2} d\bar{T}/dx \quad (13)$$

$$\bar{\rho} \bar{u}' \bar{u}' = (2/3) \bar{\rho} \bar{e} \quad (14)$$

$$\bar{\rho}' \bar{u}' \bar{T}' \approx 0 \quad (15)$$

Here $\bar{e} = \overline{u'^2}/2$ is the turbulence kinetic energy per unit mass of gas.

Equation (14) is a particular case of a more general relationship

$$\bar{\rho} \bar{u}' \bar{u}' = 2/3 \bar{\rho} \bar{e} \delta_{ij} - n_4 l \bar{e}^{1/2} (\partial \bar{\rho} u_i / \partial x_j + \partial \bar{\rho} u_j / \partial x_i) \quad (16)$$

characterizing the link between the stress tensor and the strain tensor in a turbulent flow¹²; $\delta_{ij} = 1$ at $i=j$, $\delta_{ij} = 0$ at $i \neq j$, and n_1, n_2, n_3, n_4 = dimensionless proportionality coefficients of the order of unity.

We assume in these relationships (as did the authors of Ref. 11-14) that coefficients of turbulent transfer of turbulence kinetic energy in Eq. (11), of mass in Eq. (12), and of heat in Eq. (13), are proportional to a square root of turbulence kinetic energy and to the turbulence scale l . The hypotheses [Eqs. (11-13)] are based on the assumption of conservation of individual properties (temperature, density) of a pulsating turbulent eddy prior to mixing. This means that a period of turbulence pulsation $t' \sim l/u'$ is not sufficient for the temperature of a turbulent eddy; this is true of its density as well, which changes significantly owing to the action of molecular heat conductivity. Such assumptions, regarding the turbulence, limit analysis further, as in the case in which the characteristic time of thermal relaxation $t_r \sim l^2/\sigma$ (σ is the coefficient of molecular temperature conductivity) is much

larger than a period of turbulence pulsation $t_r > t'$, i.e.

$$lu' \gg \sigma \quad (17)$$

or, according to Eq. (17), in the case of well-developed turbulence, for which the turbulent transfer coefficient lu' is much larger than the molecular transfer coefficient σ . For $\sigma \propto \nu \propto \lambda \propto \gamma \lambda$ [Eqs. (11-14 and 16)] generally are accepted, and were used in Refs. 11-14. The hypotheses about the triple correlation [Eq. (15)] are introduced because there exists a relation between pulsations of temperature and density so that, for $T' < 0$, $\rho' > 0$ and for $T' > 0$, $\rho' < 0$, respectively, the sign of the product $\overline{\rho' T'}$ is conserved when that of u' changes.

Application of approximate Prandtl's hypotheses [Eqs. (12) and (13)] imposes restrictions on the accuracy of the analysis. These hypotheses are more accurate when the profiles of \overline{T} , $\overline{\rho}$ are closest to linear ones. However, because of a nonlinear relationship between \overline{T} and $\overline{\rho}$, given by the gas state Eq. (6), the profiles of \overline{T} and $\overline{\rho}$, cannot be linear simultaneously. In addition, the inevitable nonlinearity is enhanced as pulsations of T' and ρ' are increased in comparison to \overline{T} and $\overline{\rho}$; this increases the errors introduced by Prandtl's hypotheses. Substituting Eqs. (12) and (13) into Eq. (7), and taking into account Eq. (15), we obtain an approximate gas state equation which, in contrast to Eq. (6), does not include the correlation $\overline{\rho' T'}$. This signifies that the error introduced by Prandtl's hypotheses coincides by the order of magnitude with the correlation $\overline{\rho' T'}$, so that the latter must be omitted from all of the equations.

The right-hand sides of Eqs. (8) and Eq. (9) contain the average value of an essentially nonlinear function of the reaction rate, which we shall calculate by the arithmetic mean rule.¹⁶

$$\overline{\Phi} = 0.5 [\overline{\Phi}(\overline{T} + T'_a) + \overline{\Phi}(\overline{T} - T'_a)]$$

In order to define the amplitude value of temperature pulsation included in this expression, we introduce a hypothesis

$$T'_a = l |d\overline{T}/dx| \quad (18)$$

which is a natural one, taking into account the assumption of conservation of individual properties of a turbulent eddy, upon which the Eqs. (11-13) are based.

The boundary conditions to the set of equations (2, 4, 6, 8, and 9) define the state of the flow upstream and downstream of the flame

$$x \rightarrow -\infty \quad \overline{e} \rightarrow \overline{e}_0 \quad \overline{T} \rightarrow \overline{T}_0 \quad d\overline{T}/dx \rightarrow 0 \quad \overline{u} \rightarrow \overline{u}_0 \quad d\overline{\rho}/dx \rightarrow 0 \quad (19)$$

$$x \rightarrow \infty \quad d\overline{e}/dx \rightarrow 0 \quad d\overline{T}/dx \rightarrow 0 \quad (20)$$

According to the continuity Eq. (4) and gas state Eq. (6), we obtain, for $x \rightarrow \infty$

$$\overline{u} \rightarrow \overline{u}_0 (\overline{T}_f/\overline{T}_0) \quad \overline{\rho} \rightarrow \overline{\rho}_0 (\overline{T}_0/\overline{T}_f) \quad (21)$$

Here and later in this paper the subscripts 0 and f denote the values of parameters upstream and downstream of the flame, respectively. Let us now reduce Eqs. (2, 4, 6, 8, and 9), closing hypotheses [Eqs. (11-14)], and boundary conditions [Eqs. (19-21)] to the dimensionless form; selecting, for scaling factors for \overline{e} , \overline{u} , \overline{T} , $\overline{\rho}$, and x , their characteristic values \overline{e}_0 , \overline{u}_0 , ΔT , $\overline{\rho}_0$, and x_f ; where $x_f = (\overline{e}_0^2 t_f)^{1/2}$ is the characteristic spatial scale given by the heat conductivity equation; $t_f = \rho_0/k\rho_0^2 \exp(-E/RT_f)$ is the time of the chemical reaction at a burned gas temperature and initial density and concentration of the reacting component; k is the pre-exponential factor, $\Delta T = RT_f^2/E$ is a characteristic temperature interval, R is the gas constant, and E is the activation

energy. Then, for a turbulent one-dimensional, and plane (on the average) flame, we obtain (averaging bar over dimensionless quantities is omitted)

$$\frac{d}{d\xi} \left(\rho u e - \frac{l}{P_e} \rho e^{1/2} \frac{de}{d\xi} \right) = -\frac{2}{3} \rho \frac{du}{d\xi} e + \gamma^2 \cdot P_e \frac{d\rho}{d\xi} u \frac{du}{d\xi} e^{1/2} \quad (22)$$

$$P_e \frac{d\theta}{d\xi} = \frac{d}{d\xi} \left(\rho e^{1/2} \frac{d\theta}{d\xi} \right) + \theta_0^{1-n} \langle \varphi \rangle \quad (23)$$

$$du/d\xi = (\alpha\beta/P_e) \theta_0^{1-n} \langle \varphi \rangle \quad (24)$$

$$-(l/P_e) e^{1/2} (d\rho/d\xi) + \rho u = l \quad (25)$$

$$\rho^{-1} = \alpha(1 + \beta\theta) \quad (26)$$

Here

$$\varphi(\theta) = \frac{l}{\alpha} \frac{|\Theta|^n}{1 + \beta\theta} \exp \frac{\theta}{1 + \beta\theta}$$

$$\langle \varphi \rangle = 0.5 \left[\varphi(\theta + \gamma \cdot \frac{d\theta}{d\xi}) + \varphi(\theta - \gamma \cdot \frac{d\theta}{d\xi}) \right]$$

is the averaged value of the dimensionless reaction rate; $\epsilon = \overline{e}_0^{1/2} \overline{u}_0$ is the turbulence intensity of the initial flow; n is the reaction order; $\Theta = E(T - T_f)/RT_f^2$ is the instantaneous value of dimensionless temperature

$$\theta = E(\overline{T} - T_f)/RT_f^2 \quad \theta_0 = E(\overline{T}_f - \overline{T}_0)/RT_f^2 \quad \alpha = \overline{T}_f/\overline{T}_0$$

$$\xi = x/x_f \quad P_e = \overline{u}_0 t_f/x_f \quad \beta = RT_f/E \quad \gamma = l/x_f$$

By definition of parameters

$$\epsilon = (P_e \gamma)^{-1} \quad \beta\theta_0 = 1 - \alpha^{-1}$$

The boundary conditions [Eqs. (19-21)] take the form

$$\begin{aligned} \xi \rightarrow -\infty \quad e \rightarrow 1 \quad \theta \rightarrow -\theta_0 \quad d\theta/d\xi \rightarrow 0 \\ u \rightarrow 1 \quad d\rho/d\xi \rightarrow 0 \end{aligned} \quad (27)$$

$$\xi \rightarrow +\infty \quad de/d\xi \rightarrow 0 \quad d\theta/d\xi \rightarrow 0 \quad u \rightarrow \alpha \quad \rho \rightarrow \alpha^{-1} \quad (28)$$

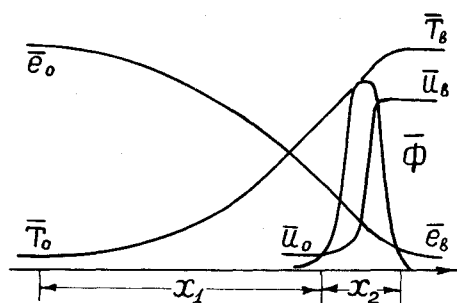
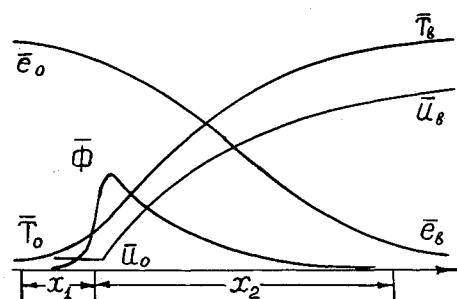
It must be kept in mind that only four of five equations in the system (22-26) are independent, since the equation for acceleration of the averaged flow [Eq. (24)] follows from the heat conductivity equation [Eq. (23)] and gas state equation [Eq. (26)].

Analysis of Interaction Between Turbulence and Flame

According to the turbulence kinetic energy balance equation [Eq. (22)] and the equation for acceleration of the averaged flow [Eq. (24)], the turbulence energy sink is located in the reaction zone, and only a spatial redistribution of turbulence energy takes place in the preheated zone of the flame, because of diffusion and convective transfer by the average flow. On the other hand, the ratio of convective to diffusion transfer of turbulence energy, mass, and heat in the reaction zone, characterized by Peclet number

$$P_{e2} = \overline{u}_0 x_2 / l \overline{e}_0^{1/2}$$

is proportional to the ratio of reaction zone depth x_2 to that of the preheated zone $x_1 \sim l \overline{e}_0^{1/2} / \overline{u}_0$. Therefore, the turbulence-to-flame interaction is determined by the structure of the flame.

Fig. 1 Flame structure for $\gamma \leq 1$.Fig. 2 Flame structure for $\gamma Pe > 1$.

The structure of a small-scale turbulent flame was investigated in Ref. 16 by computer calculations for non-compressible fluid, without taking into account turbulence energy changes in the flame. It was shown that, at $\gamma \leq 1$, the effect of temperature pulsations on the average reaction rate is small, and the structure of the turbulent flame appears similar to a laminar pattern¹⁵ (Fig. 1). As γ increases, the effect of temperature pulsations on the average reaction rate increases; therefore, the reaction begins to proceed not only at moderate temperatures close to the combustion temperature, but also at lower temperatures. This results in widening the reaction zone and in shifting the reaction rate maximum towards the initial temperature (Fig. 2).

At small γ the reaction zone is much smaller than the preheated zone

$$\xi_2/\xi_1 \sim \theta_0^{-1} \ll 1$$

therefore, in the integration of the turbulence energy balance Eq. (22), we can assume that main changes of turbulence energy, density, and temperature take place in the preheated zone. Thus, in the reaction zone, these quantities change only slightly, and are close to their values in combustion products (Fig. 1). In order to calculate turbulence energy downstream of the flame, we integrate Eq. (22) over ξ from $-\infty$ to $+\infty$, taking into account the boundary conditions Eqs. (27 and 28) and assuming ρ and e in the right-hand side of Eq. (22) to be constant and equal to their respective values in combustion products.

Thus, we have

$$e_f - I = \frac{2}{3} \rho_f e_f (\alpha - I) + \gamma^2 P_e e_f^{\frac{1}{2}} \int_1^\alpha u \frac{d\rho}{d\xi} du \quad (29)$$

Let us express the density gradient in the reaction zone, included in Eq. (29), by use of the gas state equation [Eq. (26)], via the temperature gradient in the reaction zone

$$d\rho/d\xi = -(\beta/\alpha)(1 + \beta\theta)^{-2} d\theta/d\xi \approx -(\beta/\alpha) d\theta/d\xi$$

The heat conductivity Eq. (23), written for the reaction zone without the convective term (Peclet number $Pe_2 \sim x_2/x_1$ is

small), yields, together with Eq. (24), the relationship between the temperature gradient and flow velocity

$$\rho_f e_f^{\frac{1}{2}} d\theta/d\xi \approx (P_e/\alpha\beta)(\alpha - u)$$

and hence the relationship between the density gradient and flow velocity. Calculating the integral over flow velocity in Eq. (29), we obtain the relationship determining turbulence energy in the reaction zone and downstream of the flame

$$e_f \approx \frac{3\alpha}{5\alpha - 2} \left(1 - \frac{f_1(\alpha)}{\epsilon^2}\right) \quad f_1(\alpha) = \frac{\alpha^3 - 3\alpha + 6}{6\alpha} \quad (30)$$

Equation (30) includes the rate of turbulent combustion, which in turn depends on turbulence energy in the reaction zone. Discovering this dependence reduces to determining the eigenvalue of the parameter P_e in the heat conductivity Eq. (23), with boundary conditions for temperature [Eqs. (27) and (28)]. In order to integrate the heat conductivity Eq. (23), we use Zeldovich's method,¹⁵ based on narrowness of the reaction zone as compared to that of the preheated zone (considering only the case of $n = 1$):

$$P_e = (2\rho_f e_f^{\frac{1}{2}} \alpha)^{\frac{1}{2}} / \theta_0 \quad (31)$$

Resolving Eqs. (30) and (31) with respect to e_f and \bar{u}_0 , we obtain

$$e_f = (e_f)_{\infty} f^2(s) \quad f(s) = [(1 + s^4)^{\frac{1}{2}} - 1] / s^2 \quad (32)$$

$$\bar{u}_0/\bar{v}_0 = (e_f)^{\frac{1}{4}} f^{\frac{1}{2}}(s) \quad (e_f)_{\infty} = 3\alpha/(5\alpha - 2) \quad (33)$$

Here

$$\bar{v}_0 = (\bar{e}_f^{\frac{1}{2}}/t_f)^{\frac{1}{2}} (2/\alpha)^{\frac{1}{2}} / \theta_0$$

is the turbulent combustion rate, calculated without taking into account changes of turbulence energy in the flame and the effect of temperature pulsations on the average reaction rate; $s = 4\epsilon_v/\alpha$, $\epsilon_v = \bar{e}_0^{\frac{1}{2}} \bar{v}_0$ is the turbulence intensity in the initial flow, corresponding to velocity \bar{v}_0 ; $(e_f)_{\infty}$ is the turbulence energy value downstream of the flame at $s = \infty$.

Considering the case $\gamma > 1$, corresponding to the larger Peclet number, it is worthwhile to note that the chemical reaction in the turbulent flow, as in the laminar, proceeded most rapidly at the instant when the temperature differed from the maximum burned-gas temperature by not more than by one characteristic temperature interval

$$RT_f^2/E, \text{ i.e., } \bar{T}_f - RT_f^2/E \leq T \leq \bar{T}_f$$

Inasmuch the instant temperature in the turbulent flow is summed from the mean and pulsation additives, i.e.

$$T = \bar{T} \pm l d\bar{T}/dx$$

the region of the mean temperature with the intensive chemical reaction is limited by two inequalities

$$\bar{T}_f - RT_f^2/E \leq \bar{T} \pm l d\bar{T}/dx \leq \bar{T}_f$$

This indicates, in dimensionless form,

$$-1 \leq \theta(\xi) \pm \gamma(d\theta/d\xi) \leq 0 \quad (34)$$

If temperature pulsations exceed the characteristic interval, then according to Eq. (34) the reaction zone dimensions coincide by the order of magnitude with the turbulence scale $\xi_2 \sim \gamma$ (or $x_2 \sim l$), and only positive pulsations of temperature exert substantial influence

$$\varphi(\theta + \gamma d\theta/d\xi) > \varphi(\theta - \gamma d\theta/d\xi)$$

Therefore,

$$\langle \varphi \rangle = 0.5 \varphi (\theta + \gamma d\theta/d\xi)$$

In this approximation, the maximum value of the reaction rate $\varphi_m = 1/(2e\alpha)$ is achieved at

$$\theta + \gamma(d\theta/d\xi) = -1 \quad (35)$$

close to the cold boundary of the reaction zone.

Extension of the reaction zone at large γ leads to the turbulent transfer of turbulence energy, with mass and heat being much less than they are in convective transfer (Peclet number $P_{e2} \sim \gamma P_e$ is high). In this approximation, the turbulence energy-balance equation yields

$$e = (\rho u^2)^{-2/3} \left[e^{1/3} - \frac{1}{2} \gamma P_e^2 \int_1^u \left(\frac{d \ln u}{d\xi} - \frac{d \ln \rho u}{d\xi} \right) \times \frac{u^{1/3}}{(\rho u)^{1/2}} du \right]^2 \quad (36)$$

and the heat flux from the reaction zone into the preheated zone is found to be

$$d\theta/d\xi \approx \varphi_m/P_e = (2e\alpha P_e) \quad (37)$$

Here and in the following, an asterisk denotes parameters at the boundary, separating the reaction and preheated zones (Fig. 2). Calculating, from the linear heat conductivity equation [Eq. (23)], the amount of heat required to heat up the gas in the preheated zone, equalizing it to the heat flux from the reaction zone Eq. (37), and taking Eq. (35) into account to determine the position of the reaction rate maximum, we obtain

$$\bar{u}_0 = 0.066 \bar{v}_0 \cdot \gamma \{ 1 + [1 + 4(\rho e^{1/2})/\gamma g]^{1/2} \} \quad (38)$$

and

$$\theta_* = -[\theta_0 + (\rho e^{1/2})/\gamma \rho_e] / [1 + (\rho e^{1/2})/\gamma \rho_e] \quad (39)$$

where

$$g = \gamma/2e\alpha(\theta_0 - 1)$$

Equations (38) and (39) determine the rate of turbulent combustion and temperature θ_* at the upstream boundary of the reaction zone.

Equation (38) is considerably simplified if we take into account that $\rho u \approx 1$, because, at $\gamma P_e \gg 1$, the turbulent mass transfer in Eq. (25) can be neglected as compared to convective transfer. In order to calculate the remaining integral over velocities in Eq. (36), the dependence of

$$du/d\xi \sim \langle \varphi \rangle$$

on u must be known. We point out here that, at temperature pulsations exceeding a characteristic interval, Eq. (36) yields the relationship

$$d\theta/d\xi \approx -\theta/\gamma$$

This result is supported also by numerical calculations.¹⁶ On the other hand, the equation of state (26), the continuity Eq. (25), the average flow Eq. (24), and the heat conductivity Eq. (23) yield, with turbulent transfer neglected

$$u \approx \alpha(1 + \beta\theta)$$

and

$$du/d\xi \approx \alpha\beta d\theta/d\xi$$

Finally, we obtain the required dependence

$$du/d\xi \approx (\alpha - u)/\gamma$$

and, substituting it into Eq. (36), we arrive at

$$e_f = \alpha^{-2/3} [e^{1/2} - f_2(\alpha)/\epsilon]^2$$

$$f_2(\alpha) = (g\alpha^{4/3} - 12\alpha + 3)/8 \quad (40)$$

At $\gamma P_e \gg 1$, θ_* , which is calculated from Eq. (39), and, as a result, ρ_* [from Eq. (26)], are close to their initial values. Therefore, in the preheated zone of the flame, we can neglect the decrease of convective mass flux $\rho u \approx \rho_0 u_0 \approx 1$. In the framework of this approximation, the equations of turbulence energy balance and heat conductivity in the preheated zone are similar, and e_* is expressed via θ_*

$$e_* \approx |\theta_*|/\theta_0 \quad (41)$$

Discussion

It is of interest now to analyze the relationships determining turbulence energy downstream of the flame [Eqs. (32 and 40)] and the rate of turbulent combustion [Eqs. (33 and 38)]. The damping of turbulence energy in the flame is greater as the temperature difference in the flame increases and the pulsation velocity in the initial flow decreases. As pulsation velocity increases, the decrease of turbulence energy (due to the turbulent mass transfer in the accelerated field of the average flow) diminishes in comparison to the transition of energy from the pulsation motion to the averaged flow, whose value is independent of pulsation velocity. At high pulsation velocities, damping of turbulence is determined only by energy exchange between the averaged and turbulent motion.

Flame velocity is determined by turbulence energy e_* on the boundary separating the zones of reaction and heating. Therefore, at small γ , when $e_* \sim e_f$ (Fig. 1), turbulence damping exerts an appreciable influence on flame velocity, so that it reduces the turbulent flame propagation velocity. At large γ (Fig. 2), when $e_* \gg e_f$ [Fig. 2 and Eq. (38)], the effect of damping on flame velocity is small.

As was shown in Ref. 14, combustion in the induction regime, when turbulent transfer is insignificant in comparison to convective transfer,¹⁷ proceeds with complete laminarization of the flow, if the pulsation velocity of the initial flow is below a certain critical value. As follows from Eq. (32), the final value of turbulence energy in the regime of normal propagation of a turbulent flame diminishes at small γ when pulsation velocity of the initial flow decreases, but nevertheless, the flow remains turbulent. This difference is explained by the fact that, for normal combustion with $\gamma \leq 1$, the velocity of the averaged flow depends on the final value of turbulence energy; i.e., it is contrast to the induction regime in which velocity of the averaged flow is an arbitrarily prescribed parameter. As γ increases, the reaction zone

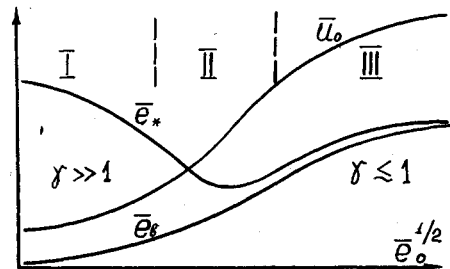


Fig. 3 Dependencies of the turbulent flame propagation rate U_0 , the turbulence energy in burned gases e_f and the turbulence energy e_* governing the flame rate upon the initial mean square pulsation velocity $e_0^{1/2}$.

widens, so that the turbulent transfer is decreased in comparison to convective one. The dependence of the flame velocity on the final value of turbulence energy e_f becomes weaker [Eq. (38)]. This regime is close to the induction regime, differing from it in that the velocity of the averaged flow is not arbitrary, but is determined by conditions of combustion [Eq. (38)]. Therefore, at large γ , complete laminarization of the flow is possible at sufficiently small pulsation velocities ($\bar{e}_0^{1/2} < f_2(\alpha) \bar{u}_0 / e^{1/2}$), as is the case for the induction regime.

Figure 3 gives a qualitative dependence of the turbulent combustion rate, turbulence energy downstream of the flame (\bar{e}_f), and turbulence energy determining the velocity of flame propagation (\bar{e}_0), on pulsation velocity of the initial flow $\bar{e}_0^{1/2}$. Three regions can be separated by the value of $\bar{e}_0^{1/2}$: 1) small velocities, $\gamma P_e > 1$; 2) intermediate velocities of the transient regime and 3) large velocities $\gamma \leq 1$. Enhancement of turbulent heat transfer with increasing $\bar{e}_0^{1/2}$ is compensated in region 1 by increased expenditures of heat on heating up the gas in the preheated zone due to changing relative dimensions of reaction and preheated zones. As a result, flame velocity becomes independent of pulsation velocity. In this region, turbulence energy damping is high, and influence of damping on flame velocity becomes small ($\bar{e}_f \gg \bar{e}_0$).

In region 2, turbulent heat transfer increases, and turbulence energy damping decreases as $\bar{e}_0^{1/2}$ rises; therefore, flame velocity in this regime depends on pulsation velocity more steeply ($\bar{u}_0 \sim (\bar{e}_0^{1/2})^m$, $0.5 < m < 1$) than is indicated in the theory disregarding damping¹⁸ ($\bar{u}_0 \sim (\bar{e}_0^{1/2})^{0.5}$).

By virtue of definition of parameters, $\epsilon = (\gamma P_e)^{-1}$. Therefore, we can extract information on the regime of turbulence-to-flame interaction not only from the parameter γ , but also from the turbulence intensity ϵ . In terms of the ϵ parameter, regime 1 ($\gamma P_e > 1$) corresponds to a low-intensity turbulence, and the regime 3 corresponds to a high-intensity turbulence.

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